Simplify $\dfrac{\cos(2x)-1}{1+\cos(x)} = \dfrac{(2\cos^2(x)-1)-1}{1+\cos(x)}$

$$= \frac{2\cos^2(x)-2}{1+\cos(x)}$$

$$= \frac{2(\cos^2(x)-1)}{1+\cos(x)}$$

$$A^2-B^2=(A+B)\cdot(A-B)$$

$$\Rightarrow \quad = \frac{2\cancel{(\cos(x)+1)}(\cos(x)-1)}{\cancel{1+\cos(x)}}$$

$$= \boxed{2\cos(x)-2}.$$

What is the meaning of

$$|b-c|<3\ ?$$

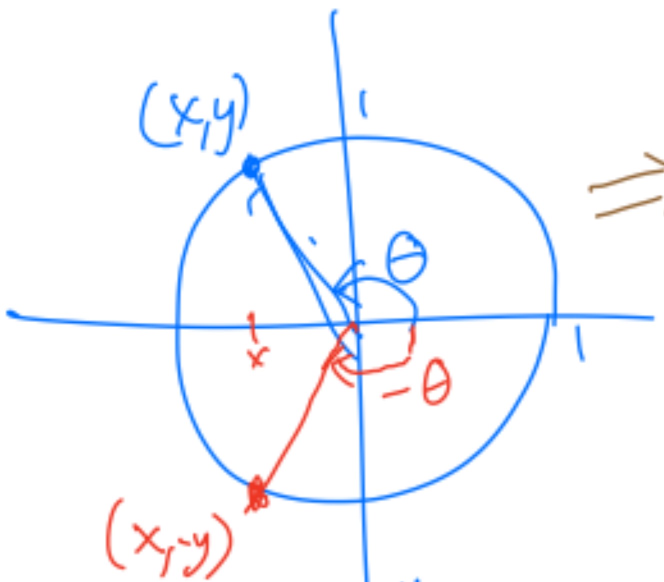
$|b-c|$ = distance between b & c.

$|b-c|<3 \Leftrightarrow$

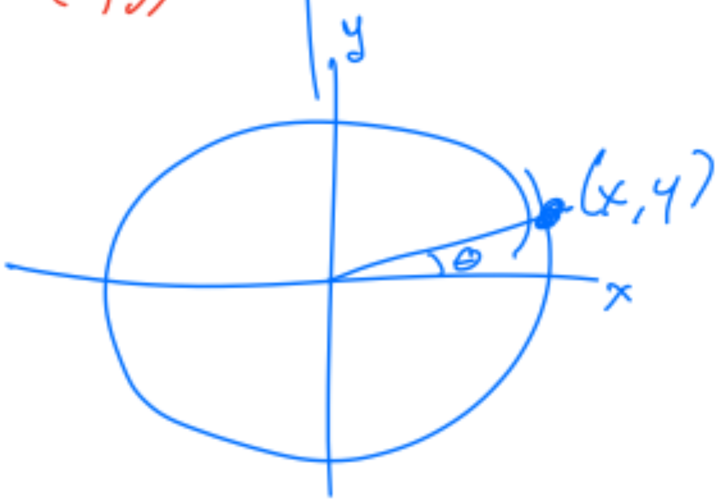
b & c are within a distance of each other.

Back to Trig ~

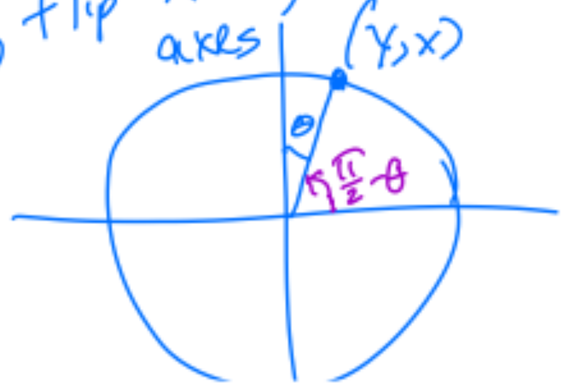
Several identities can be understood by looking at the unit circle.



$$\Rightarrow \quad \cos(-\theta) = \cos(\theta)$$
$$\sin(-\theta) = -\sin(\theta).$$



flip x & y axes



$$\Rightarrow \quad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta.$$

co-function identity.

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot(\theta)$$

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arc trig fcns – "inverse trig fcns".

arcsin(y) = angle whose sin is y

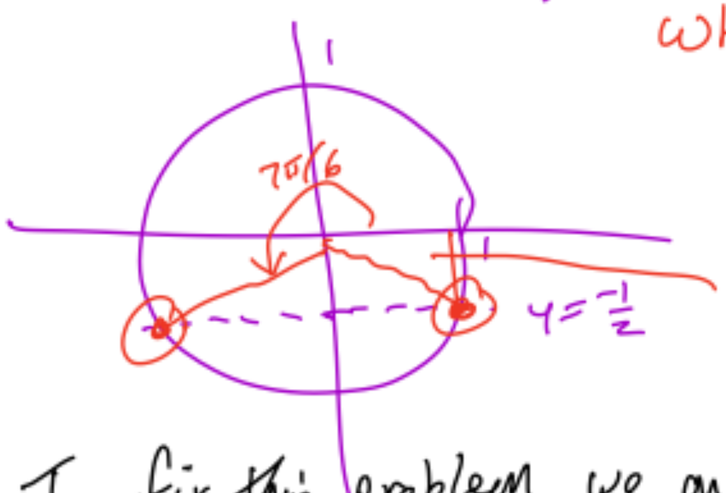
arccos(x) = angle whose cos is x

arctan(s) = angle whose tan is s.

Example

$\arcsin\left(\frac{-1}{2}\right)$ = angle whose sin is $\frac{-1}{2}$.

what is θ if $\sin\theta = \frac{-1}{2}$.



$\theta = \frac{-\pi}{6}$

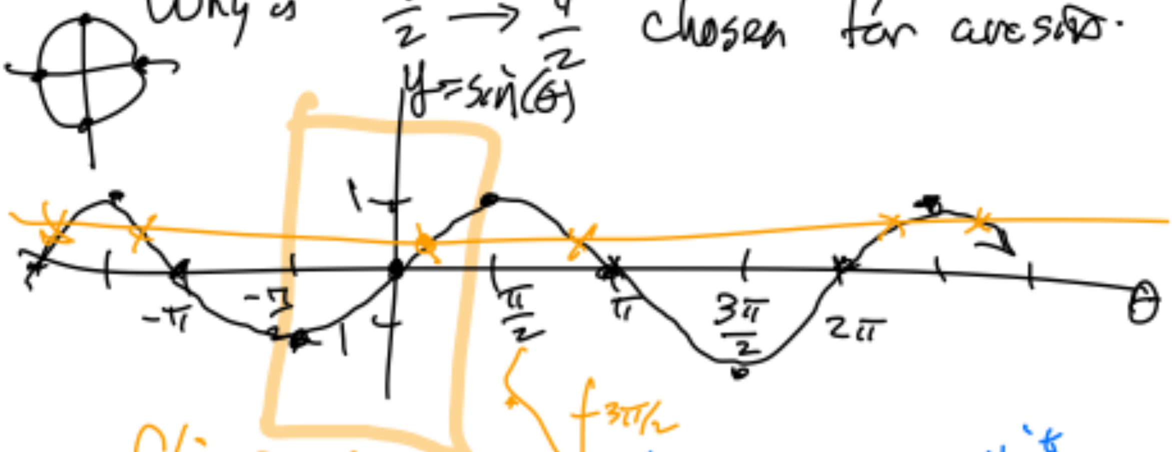
$\theta = \frac{7\pi}{6}$.

To fix this problem, we only look at the principal angles for arcsin : $-\frac{\pi}{2} \le \arcsin(y) \le \frac{\pi}{2}$.

After making this restriction, there is only one possible answer for $\arcsin(-\frac{1}{2})$.

$$\Rightarrow \boxed{\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}}.$$

Why is $-\frac{\pi}{2} \to \frac{\pi}{2}$ chosen for arcsin.



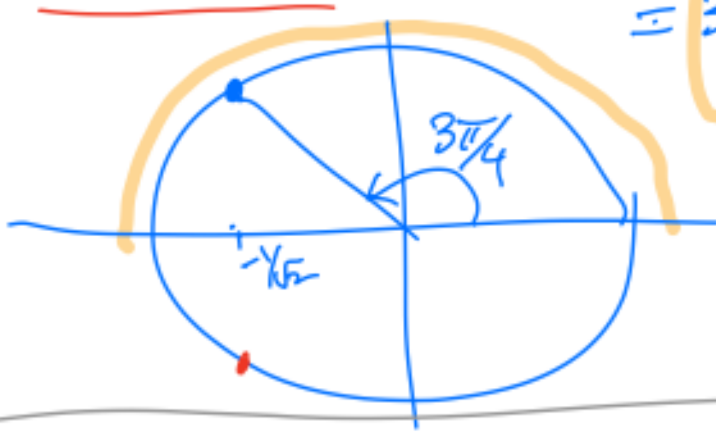
Principal ranges of "inverse" trig fcns

$$-\frac{\pi}{2} \le \begin{matrix}\arcsin(x)\\ \operatorname{arccsc}(x)\\ \arctan(x)\end{matrix} \le \frac{\pi}{2}$$

$$0 \le \begin{matrix}\arccos(x)\\ \operatorname{arcsec}(x)\\ \operatorname{arccot}(x)\end{matrix} \le \pi$$

$\arccos\left(-\frac{1}{\sqrt{2}}\right)$ = angle whose cos is $-\frac{1}{\sqrt{2}}$.

$= \boxed{3\pi/4}$.



Example: $\displaystyle\lim_{x\to 0^+} \frac{e^{-1/x}-2}{x^2+4} = ?$

$x^2+4 \to 4^+$

$-\frac{1}{x} \to -\infty \qquad \frac{1}{x} \to +\infty$

$\frac{1}{.001} = \frac{1000}{1} = 1000.$

$$e^{-1/x} = \frac{1}{\underbrace{e^{1/x}}_{\to +\infty}} \to 0^+$$

$$\therefore \lim_{x\to 0^+} \frac{e^{-1/x}-2}{x^2+4} = \frac{0-2}{4} = \boxed{-\frac{1}{2}}.$$

Example $\lim\limits_{x\to 0^-} \dfrac{e^{-1/x}-2}{x^2+4} = ?$

$$\lim_{x\to 0^-} x^2+4 = 4$$

$$x^2+4 \to 4^+$$

$$\frac{1}{x} \to -\infty$$

$$-\frac{1}{x} \to +\infty$$

$$e^{\overbrace{-1/x}^{\to +\infty}} \to +\infty \quad \text{really fast.}$$

$$\text{"} \frac{1}{e^{\underbrace{1/x}_{\to -\infty}}} \Rightarrow \frac{1}{0^+} \to +\infty$$

$$\therefore \lim_{x\to 0^-} \frac{\overbrace{e^{-1/x}-2}^{\to +\infty}}{\underbrace{x^2+4}_{\to 4}} = +\infty$$

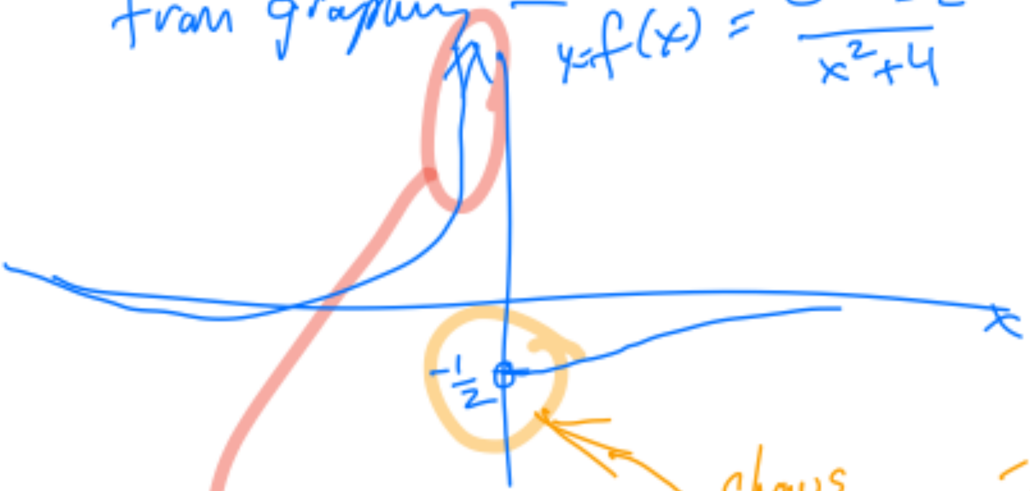
How is this possible?

$$f(x) = \frac{e^{-1/x}-2}{x^2+4}$$

$$\lim_{x\to 0^+} f(x) = -\frac{1}{2}$$

$$\lim_{x\to 0^-} f(x) = +\infty.$$

from graphing — $y=f(x)=\dfrac{e^{-1/x}-2}{x^2+4}$



shows $\lim\limits_{x\to 0^-} f(x) = +\infty$.

shows $\lim\limits_{x\to 0^+} f(x) = -\frac{1}{2}$

We say $\lim\limits_{x\to 0} f(x)$ does not exist, because $\lim\limits_{x\to 0^+}$ & $\lim\limits_{x\to 0^-}$ should exist and be the same number.

Example: Find $\lim\limits_{x\to 2^+} \dfrac{\sqrt{2}-\sqrt{x}}{x-2}$.

Solution: ① First try to plug x in (that works if the fcn is continuous.

doesn't work.. → $\dfrac{\sqrt{2}-\sqrt{2}}{2-2} = \dfrac{0}{0}$ undefined. Aaahh....

② Algebra $\dfrac{\sqrt{2}-\sqrt{x}}{x-2} = \dfrac{\sqrt{2}-\sqrt{x}}{(A+B)(A-B)}$

$A^2 - B^2$

$A = \sqrt{x}$

$B = \sqrt{2}$

$$= \frac{(\sqrt{2}-\sqrt{x})}{(\sqrt{x}+\sqrt{2})(\sqrt{x}-\sqrt{2})} = \frac{-(\sqrt{x}-\sqrt{2})}{(\sqrt{x}+\sqrt{2})(\sqrt{x}-\sqrt{2})}$$

$B - A = -(A-B)$

$$= \frac{-1}{\sqrt{x}+\sqrt{2}}$$

$$\lim_{x\to 2^+} \frac{\sqrt{2}-\sqrt{x}}{x-2} = \lim_{x\to 2^+} \frac{-1}{\sqrt{x}+\sqrt{2}} = \frac{-1}{\sqrt{2}+\sqrt{2}}$$

$$= \boxed{\frac{-1}{2\sqrt{2}}}.$$

Definition

If $f(x)$ is a function defined near a point c in the domain of f, we say that f is <u>continuous at c</u> if

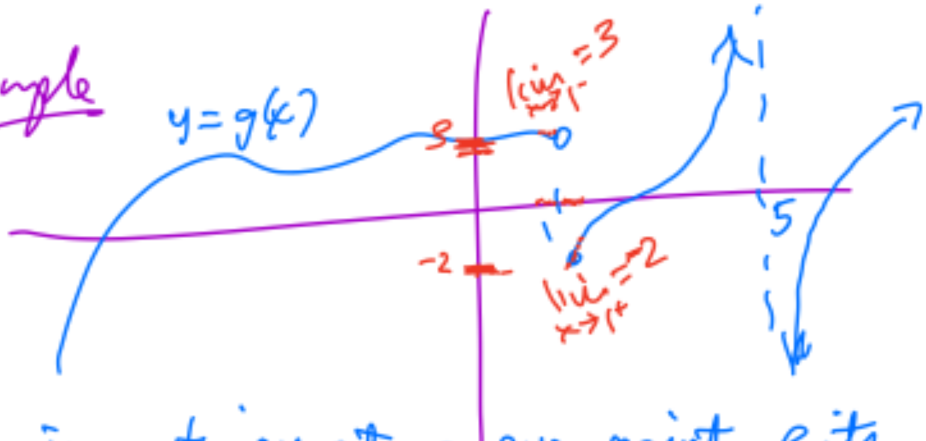
① $\lim_{x\to c} f(x)$ exists

② $\lim_{x\to c} f(x) = f(c)$

If f is continuous at every point of

its domain, we say "f is continuous".

Example



g is continuous at every point of its domain except $x = 1$.

$\lim_{x \to 1} g(x)$ does not exist

± limits don't agree

$x = 5$ is not a problem, because it is not in the domain.

Roughly: a fcn is cont. if there are no jumps in the graph in its domain.